QUIZ #8 – Solutions
Each problem is worth 5 points

15 points total

1.

a)

$$\nabla f = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x\hat{\mathbf{i}}+2y\hat{\mathbf{j}}+2z\hat{\mathbf{k}}) = -(x^2+y^2+z^2)^{-3/2}(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})$$

b)

$$\nabla \cdot \mathbf{F} = 2e^y + 0 - 2x^2y = 2(e^y - x^2y)$$

c)

$$\nabla \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2z & 12xyz & 32y^2z^4 \end{vmatrix} = (64yz^4 - 12xy)\hat{\mathbf{i}} + x^2\hat{\mathbf{j}} + 12yz\hat{\mathbf{k}}$$

2.

With parametric equations $C:\ x = 1+2t,\ y = 2,\ z = -1+6t,\ 0 \le t \le 1,$

$$\begin{aligned}\int_C (x^2+yz)\,ds &= \int_0^1 [(1+2t)^2 + 2(-1+6t)]\sqrt{2^2+0+6^2}\,dt \\ &= 2\sqrt{10}\left\{\frac{1}{6}(1+2t)^3 + \frac{1}{6}(-1+6t)^2\right\}_0^1 = \frac{50\sqrt{10}}{3}\end{aligned}$$

3.

Using x as parameter along the curve,

$$\int_C x\,dx + yz\,dy + x^2\,dz = \int_{-1}^{2} [x\,dx + xx^2\,dx + x^2(2x\,dx)] = \int_{-1}^{2} (x+3x^3)\,dx = \left\{\frac{x^2}{2} + \frac{3x^4}{4}\right\}_{-1}^{2} = \frac{51}{4}.$$